APPROXIMATE SIMILARITY CRITERION FOR AN ARC OF SELF-ADJUSTING LENGTH BURNING IN A PLASMA GENERATOR WITH VORTEX GAS STABILIZATION

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ABSTRACT: Use of the theory of similarity to analyze experimental data on arc burning in plasma generators [1–3] has made possible a qualitative jump in methods of designing such devices. The success of this approach has raised the problem of finding similarity criteria giving a good generalization for each specific plasma generator model with allowance for the corresponding arc burning characteristics.

We have obtained an approximate similarity criterion for the dimensionless potential drop along an arc burning in a plasma generator with vortex gas stabilization in the case when the length of the arc is determined by "shunting" [4]. The results of an analysis of experimental data on arcs in plasma generators with air vortex stabilization are presented.

The discovery of "shunting", i.e., closing of the arc at the electrode wall as a result of electrical breakdown of the gap between the arc and the electrode with consequent shortening of the arc [3, 7], and the investigation of its role in the process of establishment of the fall potential [4,3] make it possible to construct an idealized model of the process of arc burning in a plasma generator with vortex gas stabilization. The idealization considered below is most applicable to the two types of plasma generator with vortex gas stabilization shown in Fig. 1: the one-sided (a) and two-sided (b) configurations.

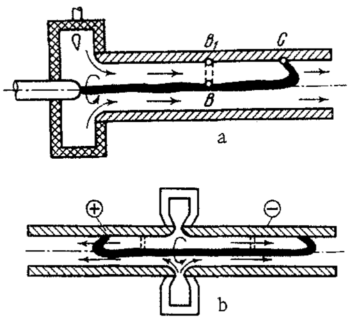

Fig. 1

These plasma generators are characterized by the presence of a single vortex chamber and "shunting" plays the chief role in establishing the arc fall potential.

Figure 2a shows an oscillogram of the arc voltage in a one-sided plasma generator, while Fig. 2b illustrates the process of buildup of the arc voltage. The instantaneous value of the arc voltage U^* may be written as

$$U^* = U_c + U_a + \int_0^l E dl.$$

Here, U_c and U_a are the cathode and anode falls, respectively, l is the length of the arc, and E the electric field strength in the column.

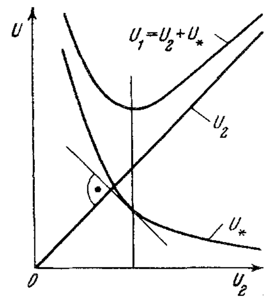

Fig. 3

When an arc is struck in the gap between the electrodes the length of the arc is minimal, which corresponds to a certain initial value of the voltage $U^* = U_0$ (Fig. 2b). As the arc is blown through the gap and its length increases, the arc voltage increases to a maximum value U_1, after which as a result of electrical breakdown between the positive column on the axis and the electrode wall and atrophy of the section BC of the column (Fig. 1a), which is bypassed by the shorter segment BB_1, the arc voltage falls to the value U_2^*, corresponding to the fall potential on the segment ABB_1. The rate of decay of the segment BC depends on the difference of the electrical resistances of segments BC and BB_1.

Denoting the mean value of the arc voltage by U and considering that the variation of arc voltage in the interval between two breakdowns is almost linear, we write

$$U = {}^1/_2\,(U_1 + U_2^*).$$

We denote by U_2 the fall of potential in the axial part of the column up to the section in which breakdown occurs; $U_2 \approx U_2^*$, since they differ only by an amount equal to the fall of potential on the segment BB_1.

The fall of potential on the segment BC is equal to U_*, the voltage at the beginning of breakdown of the gap BB_1. Then for U_1 we can write

$$U_1 = U_2 + U_*, \tag{1}$$

and the expression for the mean arc voltage takes the form

$$U \approx {}^1/_2\,(U_1 + U_2) = U_2 + {}^1/_2\,U_*. \tag{2}$$

Starting from a consideration of the arc burning conditions in a plasma generator with vortex gas stabilization and introducing the

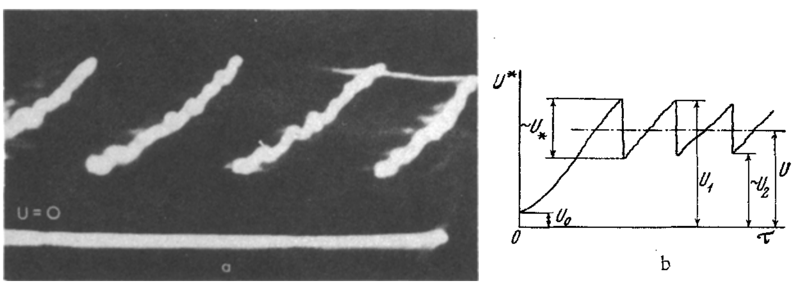

Fig. 2

breakdown equations for the case of coaxial electrodes, we can find the relation between U_2 and U_*.

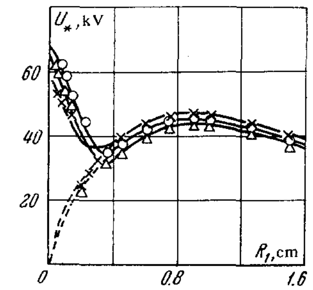

Fig. 4

For the voltage at the beginning of breakdown in the gas between coaxial cylindrical electrodes of radius R_2 and R_1 we have [8], for $R_2 \gg R_1$, the following relation:

$$\frac{U_*}{U_i \ln (R_2/R_1)} \exp\left(\frac{-A_p U_i p R_1}{U_*} \ln \frac{R_2}{R_1}\right) = \ln\left(1 + \frac{1}{\gamma}\right). \tag{3}$$

Here, R_1 and R_2 are the radii of the inner and outer cylinders, U_i is the first ionization potential of the gas, p the gas pressure, and γ Townsend's second ionization coefficient characterizing the number secondary electrons formed in the gas or at the cathode per positive ion impinging on the cathode [9], $A = 1/\lambda p$, where λ is the electron mean free path.

From the kinetic theory we can write

$$A = A_0 T_0 / T. \tag{4}$$

Here, A_0 is a quantity inversely proportional to the pressure and the mean free path at temperature T_0; T is the gas temperature at breakdown.

Taking logs in (3) and substituting for A its expression in terms of (4), we obtain

$$\frac{U_*}{U_i} = A_0 p R_1 \frac{T_0}{T} \ln \frac{R_2}{R_1} \times$$
$$\times \left\{\ln \frac{U_*}{U_i} - \ln\left[\ln \frac{R_2}{R_1} \ln\left(1 + \frac{1}{\gamma}\right)\right]\right\}^{-1}. \tag{5}$$

In the case of breakdown between the arc and the electrode wall the role of smaller cylinder (wire) is played by the arc column; therefore, neglecting heat losses in the electrode walls, the mean mass temperature of the gas outside the arc can be determined from the energy balance equation for the energy released in the arc up to the breakdown section

$$T = \frac{IU_2}{Gc_p}\theta \qquad \left(c_p = \frac{h - h_0}{T - T_0}\right). \tag{6}$$

Here, I is the arc current, G is the mass gas flow rate, c_p is the mean specific heat at constant pressure, θ is a coefficient taking into account the fact that part of the energy released in the arc is carried away by the gas flowing through the arc cross section, and h is the enthalpy of the gas.

If the temperature of the gas is not too high (for air $T \leq 5000°K$) in the breakdown section the arc occupies a small part of the electrode cross section. In this case the conditions $R_2 \gg R_1$, $\theta \approx 1$, $c_p \approx$ const are satisfied.

Substituting expression (6) in (5) and denoting R_1 by R_{arc} (arc radius), we obtain a relation between U_* and U_2:

$$\frac{U_*}{U_i} = A_0 p R_{\text{arc}} \frac{G c_p T_0}{U_i I} \ln \frac{R_2}{R_{\text{arc}}}\left\{\ln \frac{U_*}{U_i} -\right.$$
$$\left. - \ln\left[\ln \frac{R_2}{R_{\text{arc}}} \ln\left(1 + \frac{1}{\gamma}\right)\right]\right\}^{-1} \frac{U_i}{U_2}, \tag{7}$$

since the quantities p, G, R_2, T_0, and I can be assigned in advance, while the quantities U_i, A_0, γ, and c_p are constants for a particular gas. We will assume that the quantity R_{arc} is also an independent variable.

In order to obtain a second relation between U_* and U_2 we proceed as follows. We construct a graph of U_1, U_2, and U_* as functions of U_2 (Fig. 3). The curve $U_2 = f(U_2)$ is a straight line starting from the origin at an angle of 45°. The curve $U_* = \psi(U_2)$, can be constructed from Eq. (7). Using relation (1), we construct the curve $U_1 = \xi(U_2)$, which is U-shaped.

As the arc is pulled out by the gas flow, the voltage drop reaches a value corresponding to the minimum of the function $U_1 = \xi(U_2)$, after which breakdown occurs. The condition of this minimum is $\partial U_1/\partial U_2 = 0$. Differentiating expression (1) and equating the derivative to zero, we obtain a second relation between U_2 and U_*:

$$\partial U_1 / \partial U_2 = 1 + \partial U_* / \partial U_2 = 0. \tag{8}$$

If we differentiate Eq. (7) with respect to U_2, assuming that all the quantities, apart from U_2 and U_*, are fixed as U_2 and U_* vary, after certain transformations, using expression (8), we obtain the following relation containing only the breakdown voltage:

$$U_i^2 C_1 (1 + C_2 + \ln U_*/U_i) = U_*^2 (\ln U_*/U_i + C_2)^2,$$
$$C_1 = \frac{A_0 p R_a G c_p T_0}{IU_i} \ln \frac{R_2}{R_a},$$
$$C_2 = -\ln \frac{R_2}{R_a} - \ln \ln\left(1 + \frac{1}{\gamma}\right). \tag{9}$$

From Eq. (9) we can determine the value of U_* for specific values of the independent variables p, G, I, and R_2, if we know the arc radius either directly or in terms of the values of the independent variables, and then from Eqs. (7) and (2) determine the value of the mean fall of potential along the arc.

However, we do not know an analytic expression for the radius of an arc in a longitudinal flow.

Therefore, we will use the equation obtained for determining the dimensionless generalized variables—the similarity criteria defining the process of buildup of the arc voltage, with the object of utilizing them to generalize the experimental results, taking R_{arc} as the independent variable.

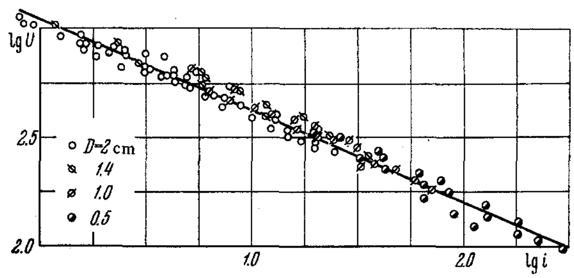

Fig. 5

For a given plasma generator model the conditions of similarity of the two electric discharges with respect to the arc fall potential are

$$K_1 = \frac{A_0 p R_a G c_p T_0}{IU_i} = \text{idem},$$
$$K_2 = \frac{R_2}{R_a} = \text{idem}, \quad K_3 = \gamma = \text{idem}.$$

Thus, the dimensionless fall potential U/U_i will be some function of these three criteria whose form is determined from an analysis of the experimental data

$$U / U_i = \varphi(K_1, K_2, K_3). \tag{10}$$

It should be noted that the quantity γ varies only slightly for the assumed electrode polarity and, moreover, in the equation γ follows two log signs; therefore, we assume that the condition γ = idem is always satisfied.

We rewrite expression (10), introducing the new criterion K_4 equal to the product of K_1 and K_2 and, assuming that similarity with respect to γ is always present, we write

$$U / U_i = \varphi_1 (K_4, K_2) \qquad \left(K_4 = \frac{A_0 p R_2 G c_p T_0}{I U_i}\right). \tag{11}$$

It can be shown that for approximate similarity it is sufficient to use only one criterion K_4 and, making use of the experimental data on breakdown between coaxial cylinders [10], to determine in advance the approximate value of the error introduced by disregarding the criterion K_2.

In Fig. 4 we present some of Uhlmann's results [10] on breakdown between coaxial cylindrical electrodes in air for a variable ratio R_2/R_1 at R_2 = const = 3 cm, where R_1 and R_2 are the radii of the inner and outer electrodes, respectively. The circles and crosses in Fig. 4, respectively, denote connection of the inner electrode to the negative and positive terminals of the source. The triangles correspond to the results obtained by superimposing 50-Hz alternating current. The data were recorded at $T = 291°$ K, $p = 1$ bar.

The dashed lines on the graph correspond to calculations using Eq. (3) and characterize the beginning of discharge formation at the point of maximum field strength (at the wall of the inner cylinder). However, it follows from Uhlmann's experiments that this discharge leads to breakdown of all the interelectrode gap only at values of $R_2/R_1 < 10$.

In fact, the variation of field strength in the gap between the electrodes is described by the equation

$$E_r = \frac{U^\circ}{r \ln (R_2 / r)}$$

(where E_r is the field strength at the variable radius r, and U° the applied voltage) and is characterized by a fall in E_r from $r = 0$ to a minimum at $R_2/r = e = 2.72$, which at $R_2/R_1 > e$ may lead to localization of the discharge at the wall of the inner electrode (to formation of a corona discharge).

From the experimental data (Fig. 4) it follows that at $10 > R_2/R_1 > e$ electron avalanches still penetrate the zone of minimum field strength and only at $R_2/R_1 > 10$ is the corona stabilized. In the latter case the breakdown voltage for the entire gap between the coaxial electrodes increases as compared with the value calculated from Eq. (3). The actual breakdown voltage for the entire gap is represented by the solid lines in Fig. 4.

We assume that the smaller cylindrical electrode is the arc column and, moreover, that the nature of the relation $U_* = \psi_1(R_2/R_1)$ is preserved. Then it is possible to neglect the effect of the criterion K_2 in functional relation (11), setting it equal to some mean (constant) value.

From the graph in Fig. 4 and the similar graphs presented in [10] for different values of radius R_2 it is clear that the error in determining the breakdown voltage over the entire interval $2 < R_2/R_1 < \infty$ should not exceed 15% U_*.

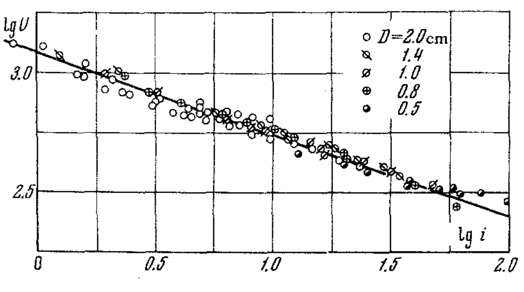

Fig. 6

Making the above-mentioned assumptions, we exclude from consideration the unknown arc radius R_{arc}. Then the condition of approximate similarity of two discharges with respect to the dimensionless fall potential will be K_4 = idem. Thus,

$$\frac{U}{U_i} \approx \varphi_1 \left(\frac{A_0 p R_2 G c_p T_0}{I U_i}\right). \tag{12}$$

It follows from Eq. (12) that if the parameters on the right side of the equation vary in such a way that the quantity in parentheses (the criterion K_4) remains constant, the arc fall potential must also

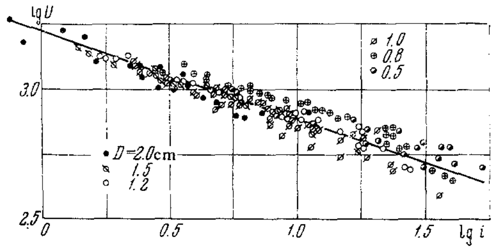

Fig. 7

remain unchanged. We present some results of an experimental verification of this conclusion for a one-sided plasma generator with an outlet electrode diameter $2R_2 = D = 2$ cm and an air exit velocity not exceeding half the speed of sound, i.e., when the pressure varies only slightly (no pressure measurements were made in this experiment):

for straight polarity

$I =$ 48	74	103	139 A
$U =$ 810	785	790	810 V

for reversed polarity

$I =$ 25	48	75	98	101	140 A
$U =$ 705	690	685	700	695	710 V

The experiment was performed for straight and reversed polarity. In this case for constancy of the criterion it was sufficient to keep the ratio G/I constant. Clearly, when the current (and the gas flow rate correspondingly) was varied by a factor of about 5, the deviations of the arc fall potential from the mean did not exceed 3% for each of the two electrode polarities. Here, by straight polarity we mean the case when the outlet electrode is the anode. For the results presented $I/G = 3.8$ A·sec·g^{-1}.

The difference in voltage on switching polarity is attributable to the different conditions of electron emission from the cathode (different values of the coefficient γ). In fact, when the arc column serves as cathode, electron emission is much facilitated as compared with the case when the cathode is the relatively cold, though strongly irradiated, wall of the electrode. The electron emission from the column may exceed that from the cold wall by some multiple of ten. However, since γ has only a weak effect (see Eqs. (5) or (9)), the difference in arc voltage is not so great and, for example, for the experimental data presented below is 10–15% (for straight polarity).

Figures 5–7 present the results of an analysis of the experimental data in U, i coordinates for air vortex stabilization of the arc in a one-sided plasma generator with straight (Fig. 5) and reversed (Fig. 6) electrode polarity and in a plasma generator with two-sided gas flow (Fig. 7) $i = I/GDp$. The data presented correspond to a variation of the gas flow rate by a factor of about 60 (from 0.5 to 30 g/sec), a variation of a current by a factor of 25 (from 10 to 250 A), and a variation of electrode diameter by a factor of four (from 0.5 to 2 cm).

Table 1 presents experimental data on the burning of the arc in a one-sided plasma generator with air as the working gas for straight electrode polarity.

The pressure in the shunting zone was everywhere close to atmospheric (it was not measured systematically). Both electrodes were made of copper.

In such an analysis using the dimensional complex $i = I/GDp$ (with physical constants discarded) in logarithmic coordinates the multitude of experimental points can be approximately represented correct to ±15–20% by a straight line, which gives the relation between the mean arc voltage and the parameters I, G, D, and p in the form of a function

$$U = C (GDp / I)^\alpha$$

Table 1

No.	G, g/sec	I, A	U, V	No.	G, g/sec	I, A	U,V
$D = 0.5$ cm				$D = 2$ cm			
1	0.62	10	226	34	4.05	48	496
2	0.62	20	158	35	4.0	58	444
3	0.63	30	142	36	4.0	80	384
4	0.63	40	135	37	4.0	99	348
5	0.63	60	123	38	4.05	121	320
6	0.92	15	253	39	4.0	141	300
7	0.93	20	228	40	4.0	160	284
8	0.91	30	190	41	7.9	60	600
9	0.92	40	176				
10	0.91	60	156	42	7.9	80	527
11	1.95	30	186	43	7.9	100	490
12	1.9	40	258	44	7.9	120	460
13	1.9	60	216	45	7.9	140	444
14	1.9	78	200	46	7.9	161	424
$D = 1$ cm				47	16	51	800
15	2.05	40	360	48	16	60	750
16	2.05	60	304	49	16	80	685
17	2.05	80	236	50	16	100	650
18	2.05	100	222	51	16	121	625
19	2.0	120	192	52	16	140	600
20	2.1	150	176	53	16	161	580
				54	16	200	556
21	4.9	40	520	55	16	250	536
22	5.0	60	396	56	31.5	50	1120
23	4.9	80	348	57	31.5	60	1030
24	5.0	100	320	58	32	79	960
25	4.9	120	306	59	32	100	870
26	4.9	150	292	60	31.5	121	830
27	7.7	40	625	61	31.5	140	815
28	7.7	50	504	62	31.5	159	800
29	7.8	62	456	63	31.5	202	770
30	7.9	78	436	64	31.5	250	755
31	7.7	99	404				
32	7.6	120	380				
33	7.8	146	344				

$$(D\ [\text{cm}],\ U\ [B],\ p[\text{bar}],\ G\ [\text{g/sec}]), \tag{13}$$

where C is some dimensional constant.

Values of C and α for the three cases mentioned are presented in Table 2, where δ is the maximum error.

Semiempirical equation (13) gives a qualitatively correct description of the relation between the mean voltage and the parameters I, G, and D. Additional unsystematized experimental data on the relation between the voltage of the self-adjusting arc and pressure qualitatively confirm the relation $U = \varphi(p)$, obtained from (13).

The magnitude of the error of the generalized relation between U and the other parameters is a consequence not only of the inaccuracy of the measurements but also of disregarding the variation of c_p and R_{arc}. The effect of disregarding changes in c_p is especially important in connection with polyatomic gases (and their mixtures) with a high dissociation energy (air, hydrogen, carbon dioxide, etc.).

If we take into account the variation of c_p with T (and at pressures below atmospheric with p) by introducing a certain approximating power function $c_p = \psi_1(T/T_0)$ for the given gas, this leads to different powers of the complexes I/G and pD in criterion K_4 (when $c_p = \psi_2(p/p_0,\ T/T_0)$ is taken into account, to different powers for p, D, and the complex I/G).

The dimensional complexes pD and I/G are residues of the dimensionless complex-criteria A_0pR_2 and Gc_pT_0/IU_i factors of the criterion K_4, that contain only independent variables.

At $p > 1$ bar, when c_p varies only slightly with pressure, the variation of c_p with T is taken into account automatically, if the experimental data are analyzed using the criteria ApR_2 and Gc_pT_0/IU_i (in this case for c_p it is necessary to take some characteristic value, for example, $c_p = c_{p_0}$ at $T = T_0$ and $p = 1$ bar) or the complexes pD and I/G. In the latter case the form of the dimensional equation will be

$$U = (I\ /\ G)^{\alpha_1}(pD)^{\alpha_2}. \tag{14}$$

Secondly, it also is possible to try to take into account the actual character of the variation of U with R_2/R_{arc}; however, this is very complicated.

Thirdly, for greater rigor it is also necessary to take into account the variation of temperature over the electrode radius and, in the course of generalization, possible dissimilarity of the temperature fields along the radius when the independent parameters vary (taking the mean mass temperature of the gas determined from Eq. (6) as the characteristic temperature presupposes such similarity).

Taking into account the second and third of the above remarks must seriously complicate the relatively simple relations obtained, but lies outside the scope of our particular investigation.

It may be pointed out that, in spite of a number of assumptions, our idealized study of the actual complicated processes that take place when a self-adjusting arc burns in plasma generators with vortex gas stabilization has made it possible to explain a series of previously obscure effects observed experimentally. These include the observed anomalies in the relation between arc voltage and electrode diameter, the different form of variation of arc length at different polarities, and the quantitative difference in the laws of variation of voltage with current for straight and reversed polarities and different gases.

It should also be noted that the generalized current-voltage characteristic (13) and (14) may be confidently used for approximate calculations of the parameters of plasma generators with a self-adjusting arc at gas temperatures at which thermal ionization of the gas outside the arc is insignificant and the chief role in breakdown is played by impact ionization (for air up to about 5000° K).

Table 2

Type of plasma generator	Polarity	C	α	δ, %
One-sided	Straight	1140	0.41	±20
	Reversed	1230	0.33	±15
Two-sided	Straight + reversed	1660	0.33	±22

A comparison of the results of generalizing the experimental data on the burning of a self-adjusting arc using the criterion K_4 obtained by the author (without allowing for the variability of c_p) and the generalization of the same experimental data using the criteria obtained in [1, 3] on the basis of an analysis of the heat transfer between the arc and the gas shows that the error of an analysis based only on the one parameter I^2/GD [1] is greater than in our case and that in a generalization based on the criterion I^2/GD it is also necessary to take into account the effect of diameter and pressure through other parameters. When the three criteria proposed in [3] are employed, the error is somewhat lower than in the case considered.

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